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ANALYSIS OF THE ORBIT OF 1966-63A NEAR 15TH-ORDER RESONANCE

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by

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A. N. Winterbottom

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R O Y A L   A I R C R A F T   E S T A B L I S H M E N T

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SUMMARY

The orbit of 1966-63A has been determined at 32 epochs between October 1976 and April 1977, using the RAE orbit refinement program PROP and some 1500 US Navy observations.

The orbit passed through exact 15th-order resonance on 1976 December 8 and the 32 values of inclination have been analysed to obtain lumped 15th-order geopotential coefficients,

$$10^9 \bar{C}_{15}^{0,1} = 36900 \pm 9700 \quad \text{and} \quad 10^9 \bar{S}_{15}^{0,1} = 12200 \pm 1700 \ .$$

When these values are used (with 22 other pairs of values) to solve for individual harmonic coefficients, there is a definite improvement over any previous solution.

It was not possible to successfully analyse the values of eccentricity as the effects of solar radiation pressure were too great and the actual change due to resonance very small.

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1   INTRODUCTION

The satellite 1966-63A, otherwise known as orbiting vehicle 1-8 (OV1-8), was launched on 14 July 1966. It was a light 'spherical skeleton', mass 10.4 kg, and had a diameter of 9.14 m. The initial apogee and perigee heights were 1013 km and 998 km, and the inclination was 144.27°. The satellite was in orbit for 11½ years, decaying on 4 January 1978 [1].

The orbit of 1966-63A had an initial period of 105.12 minutes and slowly contracted under the influence of drag to pass through 15th-order resonance in December 1976. A satellite experiences 15th-order resonance when the track of the satellite over the Earth repeats after 15 revolutions, and if the decay rate is slow enough, an orbit passing through the resonance can be appreciably perturbed by the effects of 15th-order harmonics in the geopotential. Recently 23 resonant orbits, at various inclinations, have been analysed to determine harmonic coefficients of order 15 and degree 15, 16, 17, ... 35 [2]. The satellite 1966-63A was used in this analysis, the lumped harmonic coefficients being obtained from the analysis of 25 values of inclination from US Navy orbits kindly supplied by the Naval Research Laboratory. The fit of the theoretical curve to the values of inclination was not too satisfactory, see Fig 18 of Ref 2, and the analysis of the eccentricity was not attempted as the values were not considered accurate enough. The aim of this Report is to compute more accurate and if possible more frequent orbits over the resonant period using the raw observations supplied by the US Navy and the RAE orbit refinement program PROP in the PROP6 version [3].

The orbits determined should yield more accurate lumped coefficients from the analysis of inclination and possibly allow an analysis of the eccentricity. This satellite will also serve to test the PROP program in determining orbits of higher inclination. The highest so far determined using PROP has been 98.68° [4]. Finally it may also test the accuracy of the method for restoring the odd zonal harmonic oscillation which is removed by the US Navy from their values of eccentricity[5,6], and so help future analysis.

2   THE OBSERVATIONS AND ORBITS

The orbit of 1966-63A has been determined at 32 epochs from 1481 observations between 3 October 1976 and 29 April 1977. The observations used were Navspasur observations supplied by the Naval Research Laboratory, and they have a topocentric accuracy of about 2 minutes of arc. These were the only observations available, as this satellite was not placed on the priority list for visual observers, because its inclination, 144°, meant that it only reached 36° latitude and was therefore not easily visible from the United Kingdom.

The orbits were determined at approximately weekly intervals and the orbital elements at each of the 32 epochs are listed in Table 1, with the standard deviations below each value. The epoch for each orbit is at 00 hours on the day indicated, and the PROP program fits the mean anomaly M by a polynomial of the form

$$M = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + M_4 t^4 + M_5 t^5 \ ,$$

(1)

Table 1

Values of the orbital parameters at 32 epochs with standard deviations

	MJD	Date	a	e	i	Ω	ω	ω+M₀	M₁	M₂	M₃	M₄	e	N	D	a(1-e)
1.	43054	1976 Oct 3	7023.1551 (6)	0.00367 (2)	144.1607 (9)	350.540 (3)	48.1 (3)	102.919	5308.4348 (7)	0.0883 (2)	0.00112 (4)	-	0.76	62	10.0	6997.38
2.	43062	" Oct 11	7021.8123 (5)	0.00378 (2)	144.1596 (6)	36.725 (2)	103.3 (4)	161.471	5309.9570 (7)	0.0997 (2)	0.00097 (4)	-	0.46	47	7.9	6995.27
3.	43070	" Oct 19	7020.1462 (9)	0.00322 (3)	144.1568 (9)	82.937 (4)	157.0 (3)	233.226	5311.8468 (10)	0.1239 (2)	-0.00289 (7)	-	0.57	50	8.3	6997.54
4.	43079	" Oct 28	7018.5521 (7)	0.00237 (1)	144.1560 (6)	134.966 (2)	233.1 (5)	241.704	5313.6560 (8)	0.0819 (2)	-0.00130 (11)	-	0.44	51	6.3	7001.92
5.	43087	" Nov 5	7017.5609 (5)	0.00237 (1)	144.1564 (6)	181.242 (2)	315.7 (4)	341.030	5314.7813 (6)	0.0456 (1)	-0.00156 (4)	-	0.38	46	8.4	7000.93
6.	43094	" Nov 12	7017.0993 (6)	0.00281 (2)	144.1570 (6)	221.750 (2)	17.2 (2)	163.387	5315.3054 (7)	0.0428 (4)	-	-	0.41	40	4.4	6997.38
7.	43100	" Nov 18	7016.6565 (4)	0.00320 (2)	144.1565 (6)	256.486 (2)	61.4 (4)	65.625	5315.8084 (4)	0.0465 (3)	-	-	0.44	43	5.4	6994.20
8.	43106	" Nov 24	7016.1178 (7)	0.00329 (1)	144.1566 (7)	291.219 (2)	103.9 (3)	331.209	5316.4204 (8)	0.0564 (3)	0.00253 (11)	◦	0.42	43	6.4	6993.03
9.	43114	" Dec 2	7015.1649 (4)	0.00288 (2)	144.1557 (7)	337.554 (2)	161.5 (2)	92.235	5317.5033 (5)	0.0600 (3)	-	-	0.46	45	6.3	6994.96
10.	43121	" Dec 9	7014.3800 (7)	0.00238 (2)	144.1561 (7)	18.112 (2)	223.3 (3)	294.852	5318.3954 (6)	0.0811 (3)	-	-	0.44	40	5.3	6997.69
11.	43126	" Dec 14	7013.7172 (5)	0.00230 (1)	144.1574 (6)	47.098 (2)	273.8 (7)	289.621	5319.1489 (5)	0.0624 (5)	-	-	0.36	41	3.9	6997.59
12.	43130	" Dec 18	7013.2727 (5)	0.00244 (2)	144.1590 (6)	70.289 (2)	311.4 (5)	359.841	5319.6542 (9)	0.0845 (5)	-	-	0.31	39	3.2	6996.16
13.	43134	" Dec 22	7012.8187 (5)	0.00270 (2)	144.1572 (5)	93.487 (2)	346.6 (5)	72.303	5320.1707 (8)	0.0531 (4)	0.00198 (20)	-	0.36	40	4.9	6993.88
14.	43140	" Dec 28	7012.2066 (19)	0.00324 (3)	144.1553 (10)	128.291 (2)	31.7 (4)	4.517	5320.8672 (22)	0.0369 (16)	0.00885 (56)	0.00420 (27)	0.64	42	5.2	6989.49
15.	43145	1977 Jan 2	7011.6883 (13)	0.00350 (3)	144.1562 (7)	157.307 (3)	64.8 (4)	11.761	5321.6844 (15)	0.0575 (5)	-0.00296 (43)	-	0.43	39	4.3	6986.95
16.	43152	" Jan 9	7010.8845 (13)	0.00363 (1)	144.1572 (6)	197.937 (2)	111.4 (3)	243.315	5322.3715 (5)	0.0292 (4)	-0.00180 (5)	0.00031 (2)	0.37	43	8.4	6986.84
17.	43160	" Jan 17	7010.5037 (8)	0.00289 (2)	144.1610 (6)	244.386 (2)	167.2 (2)	49.463	5322.8047 (9)	0.0220 (2)	-0.00078 (15)	-	0.39	51	5.4	6990.24
18.	43165	" Jan 22	7010.3041 (4)	0.00240 (3)	144.1620 (5)	273.422 (2)	208.0 (3)	64.906	5323.0318 (5)	0.0262 (5)	-	-	0.32	41	3.4	6993.48
19.	43170	" Jan 27	7009.9915 (10)	0.00200 (3)	144.1612 (5)	302.468 (2)	257.1 (7)	81.761	5323.3878 (11)	0.0447 (2)	0.00115 (18)	-	0.38	35	5.4	6995.97
20.	43177	" Feb 3	7008.9496 (7)	0.00210 (2)	144.1590 (7)	343.134 (3)	330.8 (4)	326.350	5324.5746 (8)	0.0933 (1)	-0.00156 (7)	-	0.41	38	7.3	6994.23

Table 1 (concluded)

MJD	Date	a	e	i	Ω	ω	ω+M₀	M₁	M₂	M₃	M₄	ε	N	D	a(1-e)
21. 43187	1977 Feb 13	7007.1551(7)	0.00249(1)	144.1561(7)	41.267(3)	54.7(4)	23.699	5326.6194(8)	0.1026(4)	-0.00013(6)	0.00030(3)	0.41	38	8.4	6989.71
22. 43193	" Feb 19	7005.9850(10)	0.00256(2)	144.1559(7)	76.171(3)	101.1(1)	356.515	5327.9534(12)	0.1152(15)	-	-	0.52	40	2.3	6988.05
23. 43198	" Feb 24	7005.2849(3)	0.00244(2)	144.1563(6)	105.269(3)	141.4(1.1)	39.411	5328.7517(12)	0.0748(15)	-	-	0.40	34	6.3	6988.19
24. 43206	" Mar 4	7004.4153(6)	0.00196(3)	144.1586(10)	151.851(3)	212.6(5)	259.205	5329.7435(7)	0.0593(3)	-	-	0.59	41	6.8	6990.69
25. 43213	" Mar 11	7003.3326(13)	0.00213(3)	144.1592(8)	192.635(3)	288.7(9)	188.420	5330.9788(14)	0.1184(4)	-0.00289(26)	-	0.64	51	5.3	6988.42
26. 43220	" Mar 18	7002.3412(6)	0.00289(2)	144.1560(7)	233.441(3)	355.0(2)	127.283	5332.1109(7)	0.0532(1)	-0.00129(5)	-	0.44	39	8.4	6982.10
27. 43228	" Mar 26	7001.7740(4)	0.00384(1)	144.1573(5)	280.082(3)	53.4(2)	12.614	5332.7583(4)	0.0346(3)	-	-	0.43	50	5.3	6974.89
28. 43234	" Apr 1	7001.3933(5)	0.00422(1)	144.1613(8)	315.075(3)	91.4(5)	19.692	5333.1927(5)	0.0380(3)	-	-	0.65	55	5.4	6971.85
29. 43241	" Apr 8	7000.5118(8)	0.00411(4)	144.1623(13)	355.916(5)	137.2(6)	332.244	5334.1996(10)	0.1161(5)	-	-	0.84	42	5.4	6971.74
30. 43248	" Apr 15	6999.3140(13)	0.00371(4)	144.1616(9)	36.790(4)	186.7(3)	294.192	5335.5684(14)	0.1039(3)	0.00363(18)	-	0.68	51	6.3	6973.35
31. 43255	" Apr 22	6997.6448(18)	0.00339(2)	144.1610(11)	77.680(4)	241.0(6)	267.078	5337.4768(21)	0.1175(9)	-0.00585(39)	0.00121(12)	0.62	38	6.4	6973.92
32. 43262	" Apr 29	6996.2768(9)	0.00321(3)	144.1598(13)	118.594(5)	296.0(1.0)	252.721	5339.0418(10)	0.1061(6)	-	-	0.98	57	5.4	6973.82

Key: MJD   modified Julian day
      a   semi major axis (km)
      e   eccentricity
      i   inclination (deg)
      Ω   right ascension of node (deg)
      ω   argument of perigee (deg)

M_0 mean anomaly at epoch (deg)
M_1 mean motion, n (deg/day)
M_2,M_3,M_4 additional coefficients in polynomial for M
ε   measure of fit
N   number of observations used
D   time coverage of observations (days)

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where t is the time measured from epoch and the number of M-coefficients used depends on the drag. For 1966-63A, which was in a nearly circular orbit at a height of about 600 km, $M_0 - M_2$ were the only coefficients required on 14 orbits; another 14 orbits required M_3 to be added; and four orbits needed $M_0 - M_4$. The value of ϵ , the parameter indicating the measure of fit, was between 0.31 and 0.98, having an average value of 0.50. The observations were therefore fitted in a satisfactory manner on all the orbits.

The values of inclination, i , had standard deviations varying between 0.0005° and 0.0013°, the average being 0.0007° corresponding to about 80 m in distance. The standard deviations for eccentricity, e , ranged between 0.00001 and 0.00004, the average value being 0.00002, equivalent to 130 m in distance.

The values of inclination from the 32 epochs in Table 1 have been plotted as circles in Fig 1 and a connecting line has been drawn through the points. The values of inclination from the US Navy orbits over the same period are plotted as crosses. The two sets of values show the same trends, the PROP values being slightly higher than the US Navy values. Only two of the US Navy values differ from the line joining the PROP values by more than the sum of their standard deviations, if the usual value of 0.003° is taken for the standard deviation of the US Navy values. This result is most satisfactory and confirms the use of 0.003° as the standard deviation for the US Navy values of inclination. At this inclination the US Navy values appear to have a bias of 0.0018° relative to PROP.

3 15TH-ORDER RESONANCE

3.1 Analysis of inclination

The theoretical equation for the variation of inclination at 15th-order resonance may be written as[2]

$$\frac{di}{dt} = \frac{n}{\sin i}\left(\frac{R}{a}\right)^{15}\left[(15 - \cos i)\bar{F}_{15,15,7}\left\{\bar{C}_{15}^{0,1} \sin \phi - \bar{S}_{15}^{0,1} \cos \phi\right\}\right.$$

$$\left. + \text{ terms in }\left\{\frac{(\frac{1}{2}\ell e)^{|q|}}{(|q|)!} \frac{\cos}{\sin}(\gamma\phi - q\omega)\right\}\right] , \qquad (2)$$

where ϕ the resonance angle is given by

$$\phi = \omega + M + 15(\Omega - \nu) \qquad (3)$$

and at exact resonance $\dot{\phi} = 0$. The variations of ϕ and $\dot{\phi}$ are given in Fig 2, and the definition of all the symbols used in equations (2) and (3) is the same as in Ref 2. In equation (2) only the main term is given explicitly: the terms involving sine and cosine 2ϕ will probably not be required, as they are associated with harmonics of order 30 and should be considerably smaller than those of order 15; and with e < 0.003 during the resonance period, the e-terms will be small also.

The 32 values of inclination in Table 1 were cleared of zonal harmonic and luni-solar perturbations using the PROD[7] computer program with one-day integration steps, and the $J_{2,2}$ perturbations were also removed. The resulting values of inclination, with

the standard deviations quoted in Table 1, were then fitted with equation (2) in integrated form by the THROE[8] computer program, which removes perturbations due to atmospheric rotation and lunisolar precession of the Earth's axis. The density scale height H was taken as 78 km, appropriate to a height of 624 km, 0.21H above perigee[9], and the atmospheric rotation rate, Λ was taken[10] as 0.8 rev/day.

This first run by THROE using just the $(\gamma,q) = (1,0)$ terms as given explicitly in equation (2) produced a rather poor fit to the values of inclination. The values of lumped harmonics obtained were:

$$10^9 \bar{C}_{15}^{0,1} = 86700 \pm 15100 \qquad 10^9 \bar{S}_{15}^{0,1} = -7900 \pm 7300 ,$$

with $\epsilon = 2.1$. The values of inclination cleared of the perturbations already mentioned are plotted in Fig 3 together with the THROE fitting with $(\gamma,q) = (1,0)$. From Ref 2 it was known that the values of $10^9 \bar{C}_{15}^{0,1}$ and $10^9 \bar{S}_{15}^{0,1}$ should be of order 49000 and 19000 respectively. Further runs were tried with more coefficients and with selected relaxations in accuracy; but the extra coefficients were not determined, the lumped coefficients $\bar{C}_{15}^{0,1}$ and $\bar{S}_{15}^{0,1}$ were no nearer to the values required and there still remained a variation in inclination which had not been successfully removed or fitted.

At this point it was realized that as 1966-63A was a spherical skeleton, it would probably be affected by solar radiation pressure. Dr P. Moore at the Earth Satellite Research Unit of the University of Aston in Birmingham has developed a program for computing the effects of solar radiation pressure on the orbital elements, and kindly ran this computer program for us, so that the values of inclination could then be cleared of this further perturbation and the THROE fitting tried again.

In order to obtain the correct change in inclination due to solar radiation pressure a value of the mass/area ratio of the satellite has to be supplied. Here it was found that a mass/area ratio of 2 kg/m^2 resulted in the best THROE fitting to the values of inclination. The values of the coefficients were:

$$10^9 \bar{C}_{15}^{0,1} = 36900 \pm 9700 \qquad 10^9 \bar{S}_{15}^{0,1} = 12200 \pm 4700 ,$$

with $\epsilon = 1.3$. This final fitting and the values of inclination with all known perturbations removed are plotted in Fig 4. In this fitting the standard deviations of the values at MJD 43134 and 43140 were increased by factors of 3 and 2 respectively.

When the new value of $\bar{C}_{15}^{0,1}$ is used in place of the old value for 1966-63A in the solution for individual 15th-order harmonics[2], there is an improvement of about 1% in the fit. When the new $\bar{S}_{15}^{0,1}$ value is used, however, the improvement is more impressive: the previous value, -6000 ± 7600, had to have its standard deviation increased by a factor of 4 in order to keep its weighted residual below 1.5, whereas the new value of $\bar{S}_{15}^{0,1}$ with its standard deviation unchanged gives a weighted residual of 1.39. In the new solutions for individual 15th-order harmonics, the values do not differ from those previously obtained[2] by more than 1/10 of the standard deviations. (The new values are to be regarded as better than the old ones: but they will not be quoted here, to avoid the confusion that would be caused by a proliferation of nearly identical solutions.)

3.2 Analysis of eccentricity

The perturbations in the eccentricity due to zonal harmonics and atmospheric drag must be removed before an attempt is made to analyse the resonance effect. The air drag model within the THROE program takes no account of the day-to-night variation in density, so a correction was applied to the values of eccentricity using the method detailed in Ref 11. As solar radiation pressure had affected the values of inclination, its effect on eccentricity would be expected to be even more important, so these perturbations were also removed using the data supplied by Dr P. Moore of Aston University, with the mass/ area ratio again assumed to be 2 kg/m^2.

The values of eccentricity cleared of all known perturbations except those due to resonance are plotted as circles in Fig 5. These values of eccentricity were then fitted with equation (12) of Ref 2 in integrated form using THROE, taking $(\gamma,q) = (1,1)$ and $(1,-1)$ terms. The fitted curve given by this THROE run is shown as a full line in Fig 5. This curve did not fit the plotted points at all well ($\epsilon = 11.4$) and it was evident that there was still a perturbation in the values of eccentricity unaccounted for. This is probably because the solar radiation pressure perturbation is more complex than has been assumed and is not adequately removed by taking a constant mass/area ratio.

A further THROE run, non-fitting, with the values of lumped harmonic coefficients calculated from the values of individual coefficients of Ref 2, gave the variations shown by the broken curve in Fig 5. This curve indicates that resonance causes very little change in eccentricity, corresponding to a maximum change in distance of around 400 m, and therefore has a much smaller effect than solar radiation pressure (4 km), zonal harmonics (7 km), or air drag (basic 4 km, day-to-night 1.5 km). In view of the uncertainty of the solar radiation pressure perturbation, there seems little hope of isolating the resonance variation, which is only three times the average standard deviation of the values. So the attempt to analyse the eccentricity was abandoned.

4 COMPARISON OF PROP AND US NAVY VALUES OF ECCENTRICITY

The values of eccentricity in Table 1 have been plotted as circles in Fig 6 after the perturbations due to zonal harmonics had been removed using the PROD[7] computer program with one-day integration steps. These values of eccentricity are joined with a connecting curve. With the US Navy values of eccentricity we first have to restore that part of the odd zonal harmonic oscillation which the US Navy remove. This is done by using the ELTRAN 2 program, and should render the US Navy values equivalent to the PROP values. These adjusted US Navy values of eccentricity were then cleared of perturbations due to zonal harmonics in the same way as the PROP values and plotted as crosses in Fig 6.

In previous analyses of US Navy values of eccentricity there has often been a residual variation correlated with ω, particularly for satellites at inclinations greater than 50°. However, Fig 6 shows no sign of such an oscillation, which would have a period of about 45 days. Also, the general agreement between the US Navy and PROP values is fairly satisfactory: if the standard deviations are taken as 0.00004 and 0.00002 respectively, 16 of the 25 US Navy values differ from the curve through the PROP values by less than the sum of the standard deviations. The inclination of 1966-63A

($144.16°$) is equivalent to $35.84°$, so Fig 6 reinforces the conclusion that deficiencies in the conversions made by ELTRAN 2 are only apparent for orbits inclined at more than $50°$ to the equator.

Fig 6 also serves to show the very large variation in eccentricity even after the removal of zonal harmonic perturbations: the large increase at the end, amounting to about 9 km, is probably due to solar radiation pressure, and this is why we failed to fit the eccentricity variation in section 3.2, where a constant mass/area ratio was assumed, which gave only a 4 km change. It seems that the mass/area must have decreased in March 1977.

5   CONCLUSIONS

The orbit of 1966-63A has been determined at 32 epochs, between October 1976 and April 1977, when the effects of resonance were being felt. About 1500 observations made by Navspasur and kindly supplied by the Naval Research Laboratory were used in the determination. The 32 orbits obtained are given in Table 1, the values of inclination and eccentricity having average standard deviations equivalent to 80 m and 130 m in distance respectively.

This was the first time the PROP program had been used to determine orbits with inclination greater than $100°$. The program ran smoothly and no problems were experienced.

The 32 values of inclination, cleared of all known perturbations except those due to resonance, were fitted with a theoretical curve and the following values of lumped 15th-order coefficients were obtained:

$$10^9 \bar{C}_{15}^{0,1} = 36900 \pm 9700 \qquad 10^9 \bar{S}_{15}^{0,1} = 12200 \pm 4700 .$$

As 1966-63A was a spherical skeleton, it was found necessary, for the first time in a resonance analysis, to remove perturbations due to solar radiation pressure before the theoretical curve could be fitted. This perturbation was removed by utilising a program developed by Dr P. Moore at Aston University: the subsequent fit gave very satisfactory results, and the lumped values, $\bar{C}_{15}^{0,1}$ and $\bar{S}_{15}^{0,1}$ yielded better solutions for individual 15th-order harmonics, especially through the new value of $\bar{S}_{15}^{0,1}$.

The analysis of the values of eccentricity was disappointing, because solar radiation pressure had such a strong effect that a determination of the lumped harmonics was impossible. However, the PROP values of eccentricity were useful in showing that the US Navy values of eccentricity, after conversion with the ELTRAN 2 program, were consistent with the PROP values.

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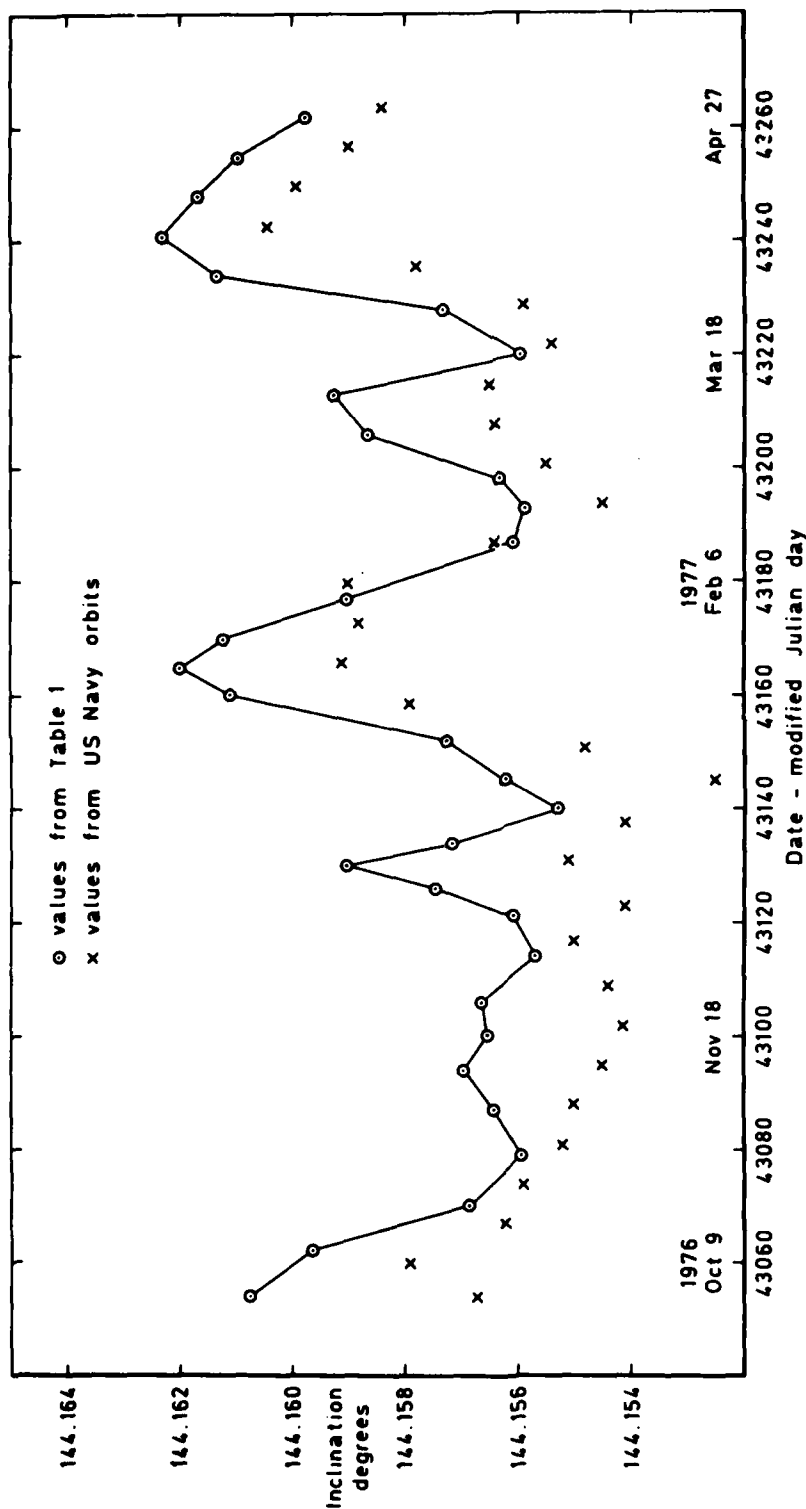| No.	Author	Title, etc
11	D.G. King-Hele	Skylab 1 rocket, 1973-27B: orbit determination and analysis. *Phil. Trans. Roy. Soc.*, A 296, 597-637 (1980) RAE Technical Report 79044 (1979)

Fig 1

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144.164

144.162

144.160

Inclination
degrees

144.158

144.156

144.154

⊙ values from Table 1
× values from US Navy orbits

1976
Oct 9

43060

43080

Nov 18

43100

43120

43140

1977
Feb 6

43160

43180

43200

Mar 18

43220

43240

Apr 27

43260

Date – modified Julian day

Fig 1   Values of inclination from Table 1 and US Navy orbits

Fig 2
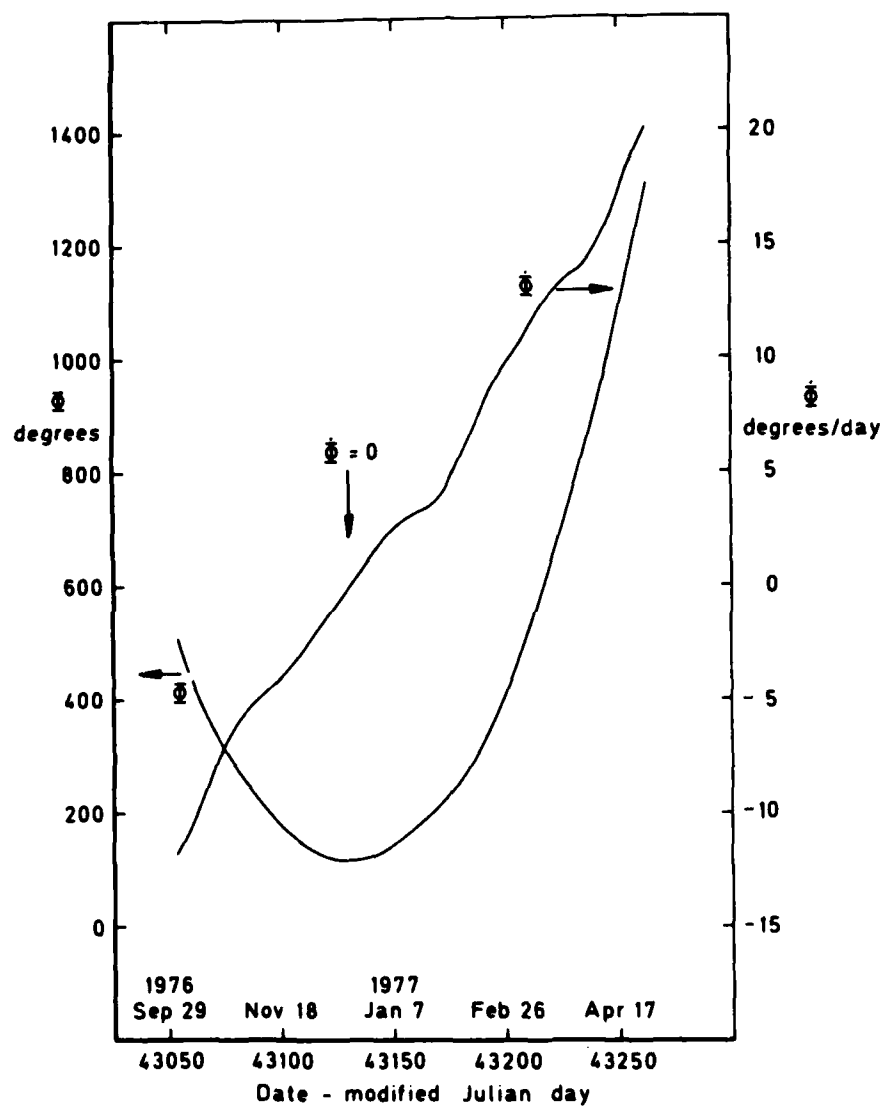

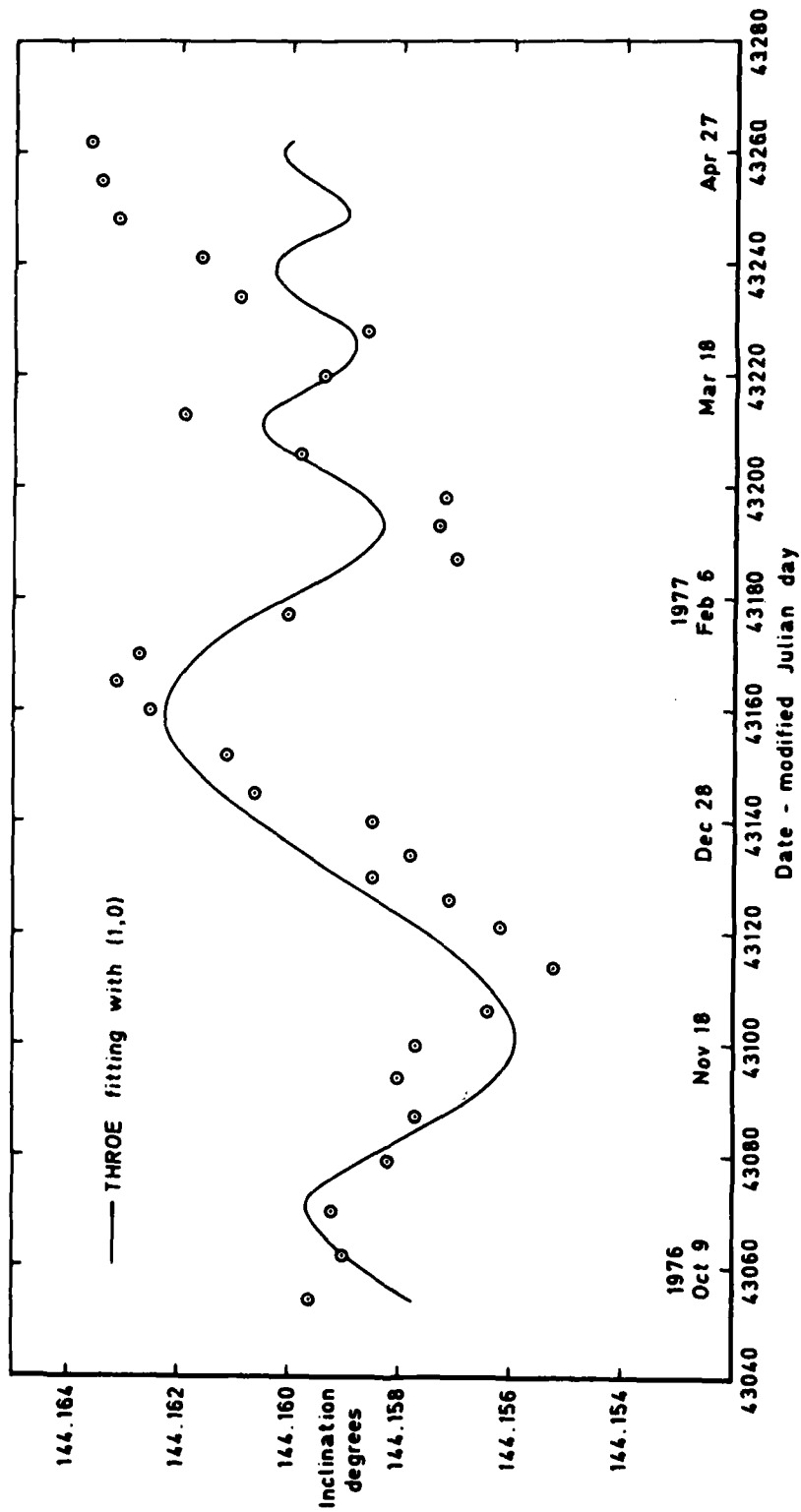Fig 2   Variation of Φ and Φ̇ near 15th-order resonance

Fig 3

Fig 3   Values of inclination, cleared of zonal harmonic, lunisolar and atmospheric rotation perturbations, and precession of the Earth's axis, with fitted curve

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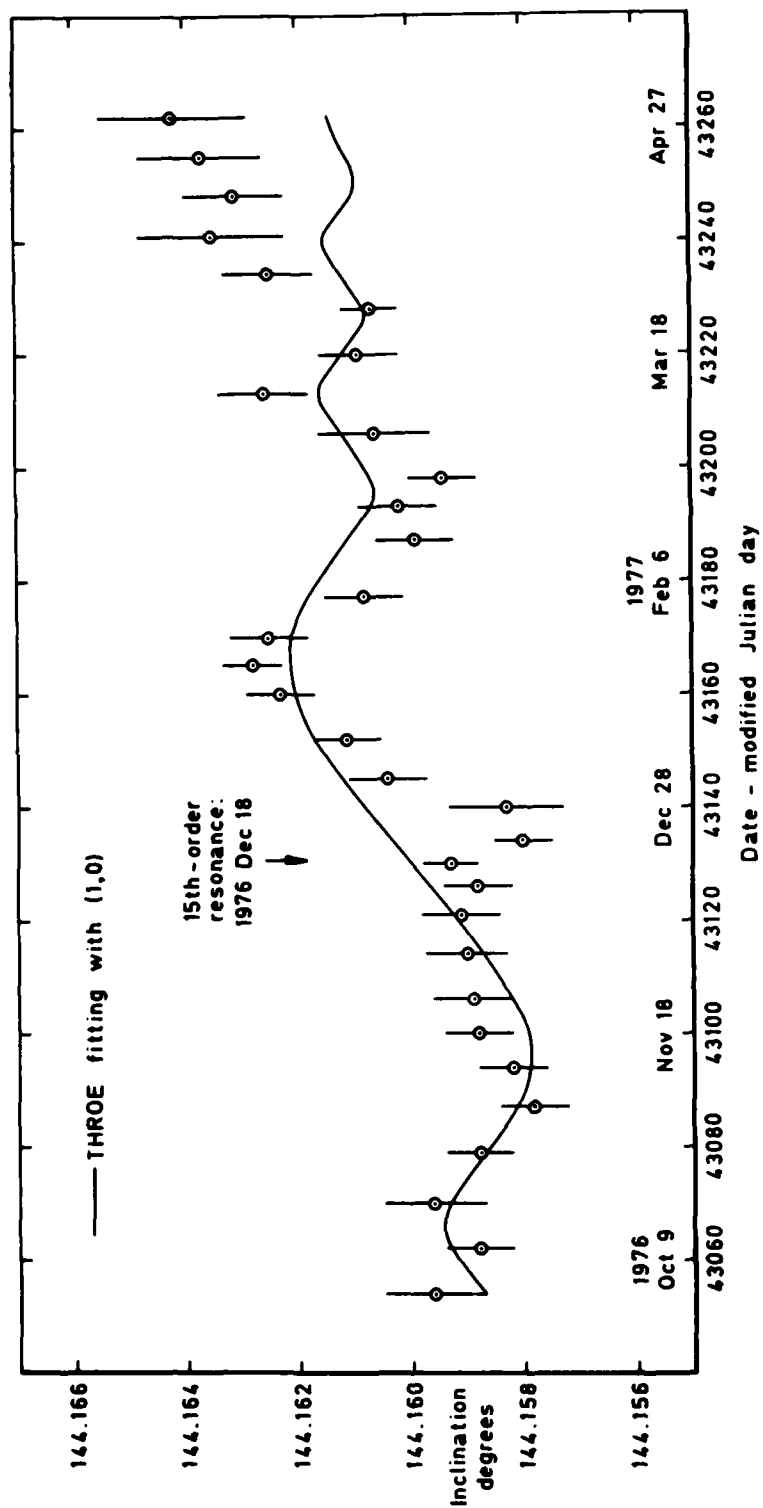
Fig 4

Fig 4   Values of inclination from Fig 3 after removal of solar radiation pressure perturbation, with fitted curve

Fig 5

The chart contains the following labels and legend:

Legend:
— THROE fitting with (1,1)(1,-1)
--- change due to resonance calculated using individual harmonic coefficients of Ref 2

Y-axis (Eccentricity): 0.0050, 0.0045, 0.0040, 0.0035, 0.0030

X-axis (Date - modified Julian day):
43060, 43080, 43100, 43120, 43140, 43160, 43180, 43200, 43220, 43240, 43260

Date labels:
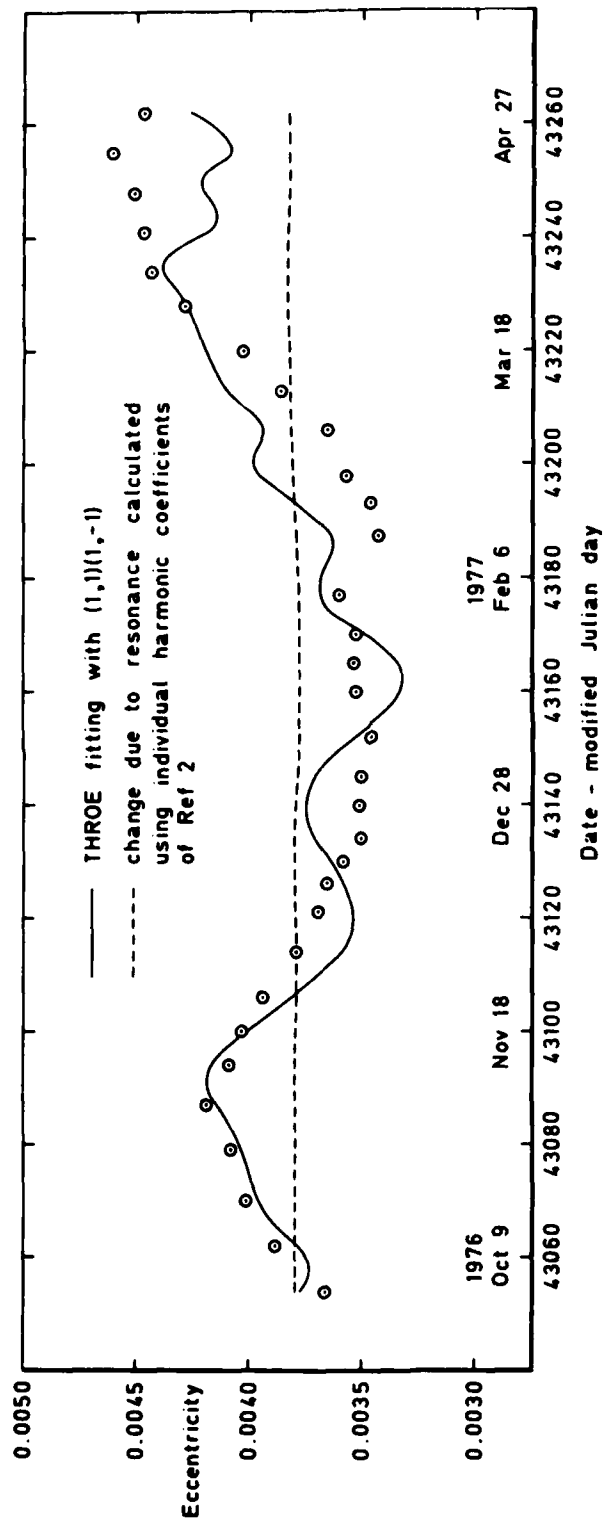1976 Oct 9, Nov 18, Dec 28, 1977 Feb 6, Mar 18, Apr 27

Fig 5   Values of eccentricity cleared of perturbations due to zonal harmonics, drag and solar radiation pressure, with fitted curve (full line) and curve predicted by individual harmonic coefficients of Ref 2

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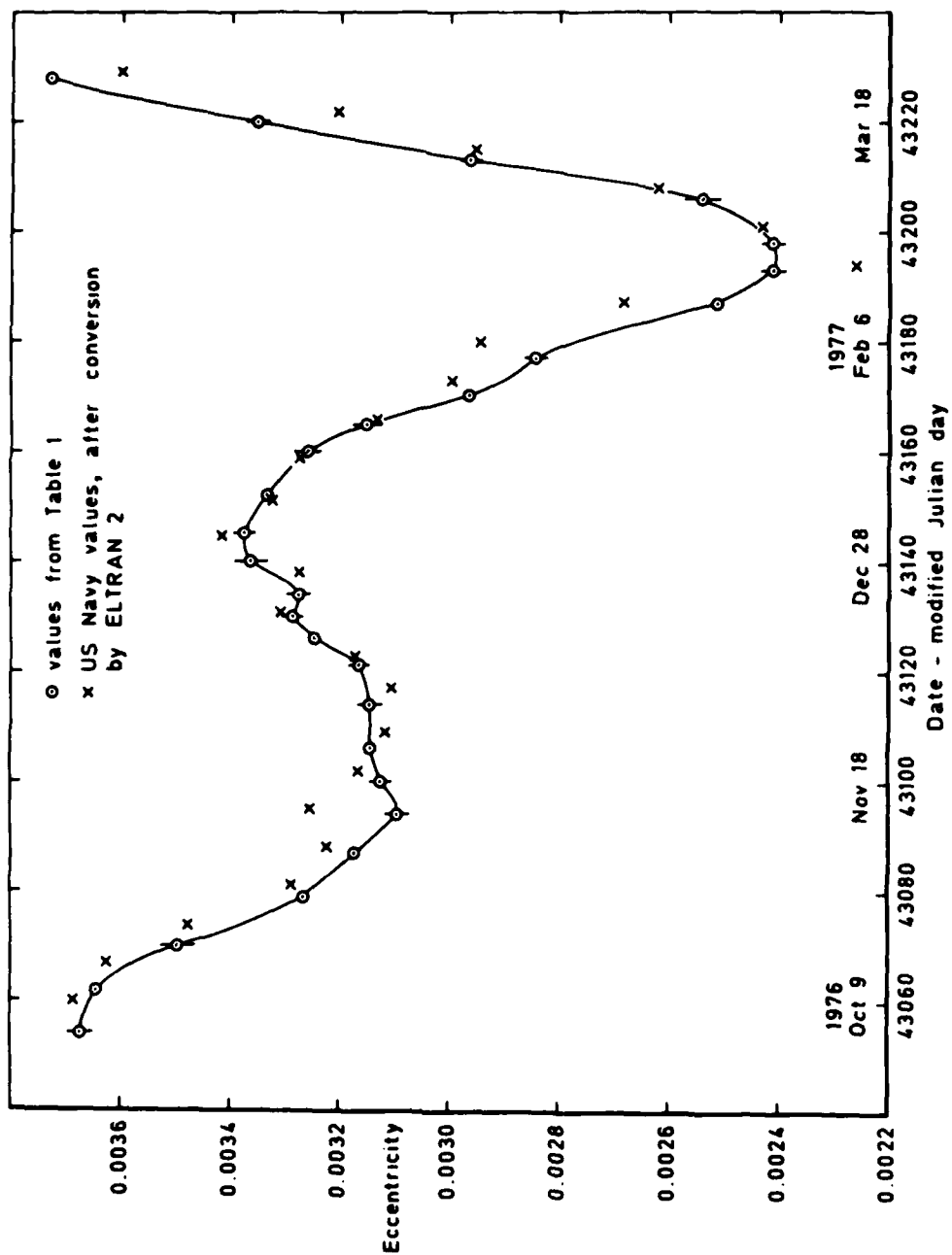
Fig 6

Fig 6   Values of eccentricity from PROP and US Navy orbits, with zonal harmonic perturbations removed

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REPORT DOCUMENTATION PAGE

Overall security classification of this page

UNCLASSIFIED

As far as possible this page should contain only unclassified information. If it is necessary to enter classified information, the box above must be marked to indicate the classification, e.g. Restricted, Confidential or Secret.

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16. Descriptors (Keywords)   (Descriptors marked * are selected from TEST)

Gravity field. Satellite orbits. Solar radiation pressure. Resonance.

17. Abstract

The orbit of 1966-63A has been determined at 32 epochs between October 1976 and April 1977, using the RAE orbit refinement program PROP and some 1500 US Navy observations.

The orbit passed through exact 15th-order resonance on 1976 December 8 and the 32 values of inclination have been analysed to obtain lumped 15th-order geopotential coefficients,

$$10^9 \bar{C}_{15}^{0,1} = 36900 \pm 9700 \quad \text{and} \quad 10^9 \bar{S}_{15}^{0,1} = 12200 \pm 4700 .$$

When these values are used (with 22 other pairs of values) to solve for individual harmonic coefficients, there is a definite improvement over any previous solution.

It was not possible to successfully analyse the values of eccentricity as the effects of solar radiation pressure were too great and the actual change due to resonance very small.

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